OBELISC: Oscillator-Based Modelling and Control using Efficient Neural Learning for Intelligent Road Traffic Signal Calculation*

Cristian Axenie[1], Rongye Shi[2](✉), Daniele Foroni[1], Alexander Wieder[1], Mohamad Al Hajj Hassan[1], Paolo Sottovia[1], Margherita Grossi[1], Stefano Bortoli[1], and Götz Brasche[1]

[1] Intelligent Cloud Technologies Lab, Huawei Munich Research Center
Riesstrasse 25, 80992 Munich, Germany
cristian.axenie@huawei.com
[2] EI Intelligence Twins Program, Huawei Cloud BU, Shenzhen, China
shirongye@huawei.com

Abstract. Traffic congestion poses serious challenges to urban infrastructures through the unpredictable dynamical loading of their vehicular arteries. Despite the advances in traffic light control systems, the problem of optimal traffic signal timing is still resistant to straightforward solutions. Fundamentally nonlinear, traffic flows exhibit both locally periodic dynamics and globally coupled correlations under deep uncertainty. This paper introduces Oscillator-Based modelling and control using Efficient neural Learning for Intelligent road traffic Signal Calculation (OBELISC), an end-to-end system capable of modelling the cyclic dynamics of traffic flow and robustly compensate for uncertainty while still keeping the system feasible for real-world deployments. To achieve this goal, the system employs an efficient representation of the traffic flows and their dynamics in populations of spiking neural networks. Such a computation and learning framework enables OBELISC to model and control the complex dynamics of traffic flows in order to dynamically adapt the green light phase. In order to emphasize the advantages of the proposed system, an extensive experimental evaluation on real-world data completes the study.

Keywords: Traffic control · Oscillator model · Spiking neural networks

1 Introduction

Road traffic congestion poses serious challenges to urban infrastructures and impacts both the social and the economic lives of people. Such fundamental reason motivated large amounts of research and systems developed to analyze, model, and control road traffic towards avoiding congestion [17]. Looking at actual technology instantiations, such as SCOOT[9], SCATS[12], PRODYN [7], or LISA [6],

* C. Axenie, R. Shi, D. Foroni, A. Wieder, M. A. H. Hassan, P. Sottovia, M. Grossi and S. Bortoli—Authors contributed equally to this research.

adaptive traffic signal control systems detect vehicles as they approach a signalized cross well in advance of the stop line. This detection, from multiple crosses, is subsequently fed into a central system, which models the flow of traffic in the area. The traffic model is then used to adapt the phasing of the traffic light signals in accordance with the flow of traffic, thus minimizing unnecessary green phases and allowing the traffic to flow most efficiently. Despite the increasing complexity of such end-to-end solutions, research on optimization is still going on. Basically, one of the differentiating aspects among the existing systems is the traffic model they use, in other words, those aspects of the physics of traffic they capture. For instance, based on large amounts of high-resolution field traffic data, work in [8] used the conditional distribution of the green start times and traffic demand scenarios for improved performance. However, high amounts of high-resolution traffic data are expensive to acquire at scale and doesn't exploit the temporal periodicity at the local level of adjacent traffic lights. Using a relatively simple model to predict arrivals at coordinated signal approaches, the work of [3] assumes nearest-neighbor interactions between signals and uses a linear superposition of distributions to optimize traffic lights phase duration. Despite finding the optimal coordination, the algorithm couldn't handle unpredictable changes to platoon shapes (i.e. occasionally caused by platoon splitting and merging) or prediction during saturated conditions (i.e. traffic jams, accidents). Hence its rather limited adaptation capabilities to disruptions that can propagate in time and space in the system.

The main goal of this study is to introduce Oscillator-Based modelling and control using Efficient neural Learning for Intelligent road traffic Signal Calculation (OBELISC), a new methodology and system for jointly modelling, learning, and controlling the dynamics of traffic flows for effective phase duration calculation. In a very good review and perspective, the study of [2], introduced the formalism of oscillator-based traffic modelling and control. Despite the good mathematical grounding, the proposed approach was static, in that it removed all convergence and self-organization dynamics of the oscillators, by replacing it with the steady-state solution. Such an approach has benefits at the single intersection level, as also the authors claim, but will fail in large-scale heterogeneous road networks (i.e. non-uniform road geometry, disrupted traffic patterns, etc.). The approach of OBELISC introduces a novel type of nonlinear coupled oscillators model based on [15], along with a nonlinear control mechanism that allows it to capture complex flow patterns and unpredictable variations [16] in large road networks. This ensures a robust control of the oscillator-based model under dynamical demand changes based on measurement of local traffic data. A similar oscillator approach was used in [13] and later in [5] as area-wide signal control of an urban traffic network. Yet, due to their complex-valued dynamics and optimization, the systems could not capture both the spatial and temporal correlations under a realistic computational cost for real-world deployment. Additionally, we are contributing with the release of a multi-cross urban traffic dataset, which contains 59 days of real urban road traffic data from 8 crosses in a city in China. Targeting a real-world deployment and superior run-time performance under real traffic flows, OBELISC is:

– using an efficient implementation in spiking neural networks [4],
– is avoiding optimization routines and operates in real-time,
– it excels in minimizing typical traffic key performance indices when the phase duration is calculated depending on the real-time demand measurements.

2 Materials and Methods

The dynamics of a traffic signal is periodic, with a phase of green - yellow - red light in one cycle, and is defined by three control parameters: a cycle length (i.e. sum of phases), a (phase) offset, and a split. The scope of our study, the phase duration calculation, is described for adjacent crosses signals in Figure 1. Typically, for a responsive traffic signal control system, adjusting the phase duration is equivalent to optimizing a given objective function (e.g. such as minimizing travel time, waiting time, or stops) in real-time, based upon perceived traffic conditions. In this section, we introduce OBELISC, as a methodology and system for jointly modelling, learning, and controlling phase calculation that exploits the periodic (i.e. oscillatory) dynamics of traffic.[3]

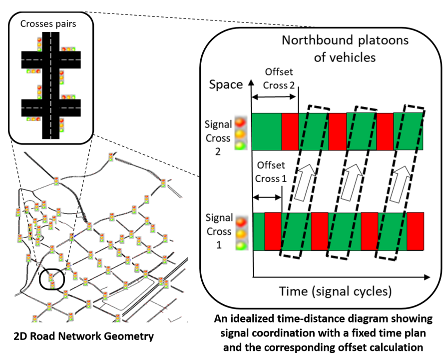

Fig. 1. Traffic light signal calculations: phase duration, offset, and cycles.

2.1 Oscillator-based Modelling of Traffic Dynamics

Traffic has a strong periodic behavior. This motivates us to describe traffic light phasing phenomenon as a repeated collective synchronization problem, in which a large network of oscillators, each representing a traffic light controlling a possible movement direction in a cross, spontaneously locks to a common operation phase. Subsequently, the phase duration adjustment factor is computed as a

[3] Codebase available at: https://github.com/omlstreaming/ecml2021

function of the oscillator time to synchronization. The intuition is the following: 1) each of the oscillators is injected with external traffic flow data impacting its local dynamics, and 2) the oscillator network converges to a steady state used to extract adaptive factor to adjust the traffic light phases. Despite the inevitable differences in the natural oscillation frequencies and injected data of each oscillator the network ensures that each of the coupled oscillators repeatedly locks phase. We extend the basic Kuramoto oscillator [10] with additional components to account for spatial as well as temporal interactions among the oscillators and an external perturbation model as described in Equation 1.

$$\frac{d\theta_i(t)}{dt} = \omega_i(t) + k_i(t)\sum_{j=1}^{N} A_{ij} sin(\theta_j(t) - \theta_i(t)) + F_i sin(\theta^*(t) - \theta_i(t)) \tag{1}$$

where:
θ_i - the amount of green time of traffic light i
ω_i - the frequency of traffic light i oscillator
k_i - the flow of cars passing through the direction controlled by oscillator i
A_{ij} - the static spatial adjacency coupling between oscillator i and oscillator j
F_i - the coupling of external perturbations (e.g. maximum cycle time per phase)
θ^* - the external perturbation (e.g. traffic signal limits imposed by law)
The model underlying OBELISC assumes that the change in allocated green time θ_i for a certain traffic light i, for a certain direction, depends on the: 1) the internal frequency of the corresponding (traffic light) oscillator ω; 2) the current flow of cars k_i in that direction; 3) the spatial coupling A_{ij} of the oscillators through the street network that weights the impact of a nonlinear periodic coupling of the oscillators $sin(\theta_j(t) - \theta_i(t))$; and 4) the external perturbation θ^* with weight F_i which ensures, for instance, that the output of the system stays in the bounds of realistic green time values imposed from the traffic laws. Given the known topological layout of the road network and the computed green times of each of the oscillators, when the dynamics converge (i.e. the solution of the differential equation 1), we infer the actual adaptive factor to be applied to the traffic light phase duration between adjacent (coupled) oscillators corresponding to adjacent moving directions. More precisely, given the steady state value of the green time (i.e. the solution $\theta_i(t_f)$), we calculate the phase duration as the time to synchronization of each oscillator relative to the ones coupled to it. From the dynamics synchronization matrix ρ at each time t the phase duration update is calculated as $\arg\max_t\{\rho(t) > \tau\}$ where $\rho_{ij}(t) = cos(\theta_i(t) - \theta_j(t))$ and $0 < \tau < 1$.
In order to ground the analytic formulation, we describe a simple, regular 5×5 lattice composed of $N = 25$ oscillators. For simplicity, in this example, each oscillator is responsible for an entire cross (i.e. the 4 adjacent directions: N, S, W, E) and the spatial coupling A_{ij} is given by the topology of the lattice, as shown in Figure 2 a. Here, each oscillator i dynamics is described by the superposition of its natural oscillation frequency ω_i and the cumulative impact of neighboring (coupled through A_{ij}) oscillators weighted by the flow of cars k_i through the cross controlled by oscillator i. The external perturbation term

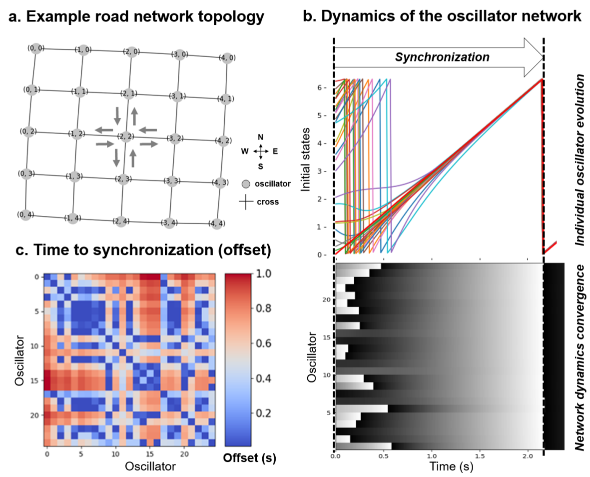

Fig. 2. Oscillator-based dynamics.

$F_i sin(\theta^*(t) - \theta_i(t))$ is neglected for simplicity. Figure 2 b describes the internal dynamics of such a network model for traffic control where given the different initial conditions of each oscillator, the coupling dynamics enforces consensus after some time (i.e. 2.1s). The steady state is then used to extract the actual phase duration by simply calculating the time to synchronization $\arg\max_x\{\rho(t) > \tau\}$, as a per oscillator relative time difference, from the ρ matrix in Figure 2 c. Here, the choice of τ determines how fast a suitable steady state is reached.

2.2 Robust Control of the Oscillator-based Networked Dynamics

The network dynamics of OBELISC (Equation 1), is judiciously parametrized to cope with the normal daily traffic profile. This can be visible in Figure 3 where the model is able to keep the lost time through a single cross to an acceptable value, around 70s (see Figure 3 b). In the case of traffic disruptions (e.g. accident, sport events, or adverse weather conditions), the system cannot capture the fast changing dynamics (i.e. steep derivatives) of the traffic flow (see Figure 3 a) and, hence, performs poorly, for instance in preserving an acceptable time loss (i.e. difference in the duration of a trip in the traffic free vs. full traffic) over rush-hour (see Figure 3b around 18:00). The example in Figure 3 illustrates a limitation of such dynamic networked models, namely robustness to uncertainty. Be it structured uncertainty (e.g. sub-optimal choice of the internal oscillator frequency ω or a sudden time varying topological coupling A_{ij} through trajectory re-routing) or unstructured uncertainty (e.g. unmodelled dynamics through the single use of $\dot{\theta}(t)$ and neglecting rate of change given by the Laplace operator $\ddot{\theta}(t)$), the system in Equation 1 is unable to converge to a satisfactory solution given input k and coupling constraints.

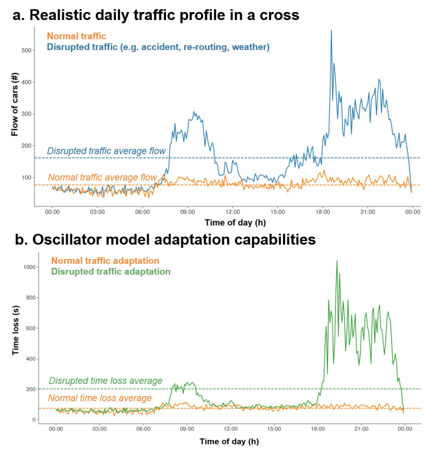

Fig. 3. Oscillator-based model dynamics adaptation capabilities.

To address this challenge, we extend Equation 1 with a robust control law. We chose to systematically maintain stability of the oscillatory dynamics by using a robust control approach which ensures consistent performance in the face of uncertainties. Sliding mode control [16]is a well established control engineering method to compensate for uncertainty and handle highly nonlinear problems. At its core it captures and controls the impact of higher-order motion (i.e. second derivative) through a high-frequency switching of the control law towards synchronization. Such a discontinuous robust control "drives", through a regularizing control law term $u(t)$, the coupled dynamics of the oscillators towards a desired dynamics (i.e. sliding surface).

$$\frac{d\theta_i(t)}{dt} = \omega_i(t) + k_i(t) \sum_{j=1}^{N} A_{ij} sin(\theta_j(t) - \theta_i(t)) + F_i sin(\theta^*(t) - \theta_i(t)) + u_i(t) \quad (2)$$

with

$$\begin{aligned}
u_i(t) &= \epsilon_1 \int_0^t \hat{s}_i(\tau) d\tau \\
\frac{d\hat{s}_i(t)}{dt} &= \epsilon_2 (\sum_{i,j} (\hat{s}_j(t) - \hat{s}_i(t)) + s_i(t)) \\
\frac{ds_i(t)}{dt} &= \epsilon_3 \sum_{j} (s_j(t) - \frac{d\hat{s}_i(t)}{dt}) - sign(\hat{s}_i(t)) \frac{d^2\theta_i(t)}{dt^2} \\
0 &< \epsilon_1 < \epsilon_2 < \epsilon_3 < 1
\end{aligned} \quad (3)$$

where:
$s_i(t)$ - the surplus energy of traffic light i oscillator
$\hat{s}_i(t)$ - the estimated surplus energy of traffic light i oscillator

The goal of the regularizing sliding mode control law u_i is to "push" the network of coupled oscillators, with a step size of ϵ, towards a dynamics which accommodates the disruptions in the flow of cars k (i.e. captured by $\ddot{\theta}_i(t)$). Intuitively, this assumes that the controller captures the second-order motion (i.e. $\ddot{\theta}_i(t)$) of the oscillator and compensates for it asymptotically until the surface is reached. This assumes, in first instance, choosing an appropriate sliding surface that minimizes the energy surplus $s_i(t)$ as illustrated in Figure4 c. Following Equation 3, the regularizing control law $u_i(t)$ applied to oscillator i is the area under the curve (i.e. the integral) of the estimated energy surplus, depicted in Figure 4 c. Interestingly, the (estimated) surplus energy, which keeps oscillator i away from the desired robust dynamics $\hat{s}_i(t)$ depends on the local oscillators interaction $\sum_{i,j}(\hat{s_j}(t) - \hat{s}_i(t))$ and the actual surplus energy. The change in surplus energy is the actual dynamics of convergence to the sliding surface and is based on the cumulative impact of neighboring oscillators $\sum_j s_j(t)$ and the Laplacean of the green time $\ddot{\theta}_i(t)$ weighted by the direction of the convergence $sign(\hat{s}_i(t))$. The property of insensitivity of sliding surface in Equation 2 to the oscillatory dynamics [4] is utilized to control the reaction of the network of coupled oscillators to uncertainty. We realized this practically by adding the regularizing term $u_i(t)$ in the local dynamics of each oscillator described by Equation 1. To get a better understanding of Equation 2, we now exemplify, in Figure 4, the impact the sliding mode controller has upon the dynamics of a road network when facing traffic disruptions from a real scenario (details about the data is provided in the Experiments and Results section). We consider a region composed of 8 crosses and $N = 29$ oscillators as described in Figure 4 a. In our case, the network of coupled oscillators is a system with discontinuous

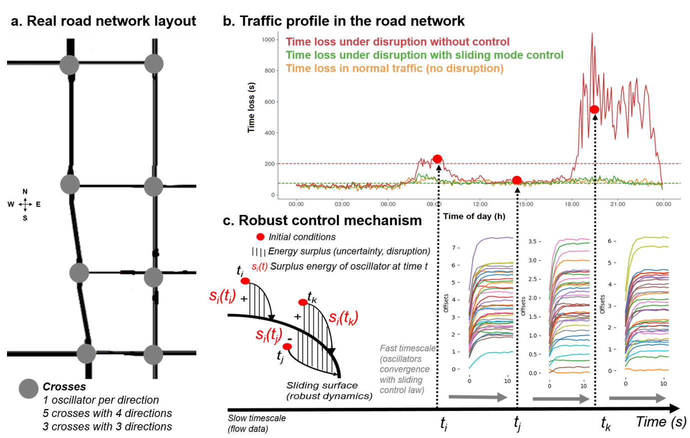

Fig. 4. Sliding Mode Control for oscillator-based model dynamics adaptation.

control (i.e. the control law $u_i(t)$ uses the sign of the energy surplus to drive

[4] For a thorough analysis of sliding modes invariance see [16].

the system towards the robust dynamics). Basically, as shown in Figure 4 b, c, given each sample of flow data $k_i(t_i...t_j...t_k)$ (from the road sensors) there is a fast convergence time-scale which allows the oscillators to reach steady state. This state is reached under sliding mode control by compensating for the disruptions in the traffic flow modelled by the second-order motion $\ddot{\theta}_i(t)$. The stationary state is subsequently probed for the actual phase duration, relative to each coupled oscillator by solving $\arg\max_x\{\rho(t) > \tau\}$ where $\rho_{ij}(t) = cos(\theta_i(t) - \theta_j(t))$ and $0 < \tau < 1$. Due to the fast changes, occurring during disruptions (see Figure 4 b - rush hour around 18h00), in the slow time-scale of traffic flow (i.e. sensory data), the network of coupled oscillators benefits from the sliding mode control law to compensate for the abrupt changes and to reach consensus, as shown in Figure 4 c - right panel. This consensus state describes the point when the system dynamics reached the sliding surface, in other words when the magnitude of the surplus energy decayed at a finite rate over the finite time interval (i.e. fast timescale in Figure 4 c - left panel). The regularization approach we propose has a simple physical interpretation. Uncertainty in the system behavior in the face of disruptions appears because the motion equations of the dynamics in Equation 1 are an ideal system model. Non-ideal factors such as unmodelled dynamics and sub-optimal parameter selection are neglected in the ideal model. But, incorporating them into the system model eliminates ambiguity in the system behavior which "slides" to a robust dynamics.

2.3 Representation, Learning, and Dynamics in Neural Networks

The notion of phase allows for a direct identification of the system's state in terms of a one-dimensional variable, described in Equation 1. This facilitates an analytic approach to robustly control such dynamics, as shown in Equation 3. Yet, such complex analytical description of networked dynamics is not tractable for large real-world deployments. In order to deploy an efficient traffic signal phase optimization with OBELISC, the data representation, the oscillatory network dynamics, and the robust controller, are implemented in the Neural Engineering Framework (NEF) [4]. NEF offers a systematic method of "compiling" high-level dynamics, such as ordinary differential equations (ODEs), into synaptic connection weights between populations of spiking neurons with efficient learning capabilities.

Representation of traffic flow data In NEF, neural populations represent time-varying signals, such as traffic flow data, through their spiking activity. Such signals drive neural populations based on each neuron's tuning curve, which describes how much a particular neuron will fire as a function of the input signal (see Figure 5 - Encoding Neural Population, upper panel). The role of the representation (i.e. complemented by a pair of operations for encoding/decoding) is to provide a distributed version of the real-valued input signal. Basically, using this representation, we can estimate the input signal originally encoded by decoding the pattern of spikes (see Figure 5 - Encoding Neural Population). The decoding weights are determined by minimizing the squared difference between the decoded estimate and the actual input signal and accounts for the weights learning process[5].

Learning arbitrary functions of flow data Encoding and decoding operations on NEF neural populations representations allow us to encode traffic flow signals over time, and decode transformations (i.e. mathematical functions) of those signals. In fact, NEF allows us to decode arbitrary transformations of the input signal. In our case the right-hand side of Equation 1 contains a non-linear combination of terms, out of which, for instance, the sinus of the relative phase difference $sin(\theta_j(t) - \theta_i(t))$ is decoded as a sinus transformation from a population encoding the phase difference $\theta_j(t) - \theta_i(t)$. The same principle applies to the robust controlled dynamics in Equation 2 and is depicted in Figure 5. This process determines how we can decode spike trains to compute linear and nonlinear transformations of the various signals encoded in a population of neurons. Essentially, this provides the means of learning the neural connection weights to compute the function between populations (e.g. product between the population encoding the spatial adjacency coupling A_{ij} and the population encoding the sinus transformation of the phase difference $\theta_j(t) - \theta_i(t)$).

Dynamics of traffic oscillator network in neural networks Fundamentally, NEF automatically translates from standard dynamical systems descriptions to descriptions consistent with neural dynamics. Using the distributed neural representation of the traffic data and learning arbitrary functions of traffic data, we can now describe the combined dynamics implementation of the network of oscillators and the sliding mode controller. Figure 5 introduces the high-level implementation details. The neural implementation in Figure 5 is bound to each oscillator i in the network. Each oscillator is fed with traffic flow data $k_i(t)$ corresponding to the direction it controls. The real-valued data is then encoded in a distributed pattern in the Encoding Neural Population. This encoding process is visible in the Spiking Activity and Neural Activation panels of Figure5, where each neuron encodes the input data in a frequency modulated train of spikes (Spiking Activity). The temporal activation of each of the encoding neurons relative to each other is illustrated in the Neural Activation panel. As one can see, in Figure 5 - left and low-left panels, the decoded flow if cars is a noisy version of the actual input (intuitively, more neurons will provide a better reconstruction but more computational cost). The encoded traffic flow data is then fed to the actual combined dynamics (i.e. oscillator network and sliding mode controller) in the Robust Controller OBELISC Population. This neural population has a recurrent connection that implements the dynamics of the right-hand side of Equation 2. More precisely, this population splits the Equation 2 in terms and realizes each multiplication, nonlinear function, and summation in separate connected populations. Basically, the population encoding the oscillation frequency

[5] For a thorough overview of practical Neural Engineering Framework (NEF) see [4].

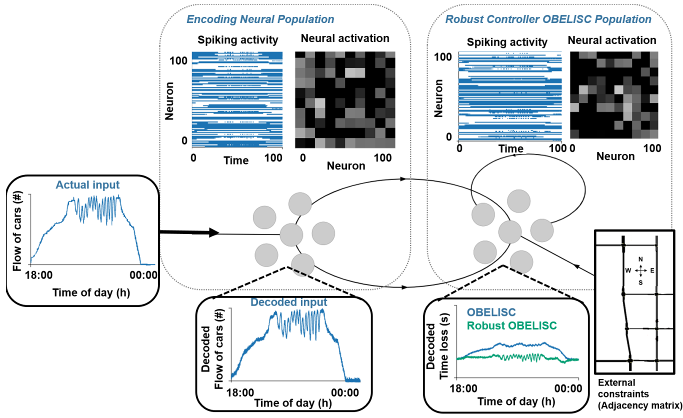

Fig. 5. Representation, Learning, and Dynamics of Robust Oscillator Network

ω_i will be connected through a sum function to the population encoding the sum of the external constraints (e.g. A_{ij} and F_i) weighting the phase differences $\theta_j(t) - \theta_i(t)$, both decoded from separate neural populations implementing the product and sinus functions. These operations implemented in neurons correspond to $A_{ij}sin(\theta_j(t) - \theta_i(t)) + F_i sin(\theta^*(t) - \theta_i(t))$. In order to visualize the benefit of the sliding mode control in the overall dynamics, we also compute the time loss, as a simple metric, in the Robust Controller OBELISC Population. As previously mentioned, the sliding mode controller makes a trade-off between performance and control activity (i.e. better performance in terms of time loss under faster switches of the control law). Basically, this is visible in Figure 5 - low right panel, between 18:00 and 24:00, where the OBELISC oscillator network dynamics performs smoother but worst in optimizing the time loss, whereas the Robust OBELISC (i.e. dynamics containing the sliding mode regularization) improves the time loss with the price of high-frequency low-amplitude oscillations.

3 Experiments and Results

The experiments and evaluation use the SUMMER-MUSTARD (Summer season Multi-cross Urban Signalized Traffic Aggregated Region Dataset) real-world dataset, which contains 59 days of real urban road traffic data from 8 crosses in a city in China[6]. The road network layout underlying is depicted in Figure 4 a. In order to perform experiments and evaluate the system, we used the real-world traffic flows in the Simulator for Urban Mobility (SUMO)[11]. This realistic vehicular simulator generates routes, vehicles, and traffic light signals that reproduce the real car flows in the dataset.

In our experiments, we comparatively evaluated the adaptive behavior of OBELISC and relevant state-of-the-art approaches, against the static traffic

[6] We release the SUMMER-MUSTARD real-world dataset used in OBELISC experiments at: http://doi.org/10.5281/zenodo.5025264.

planning (i.e. police parametrized phases), used as baseline. We performed an extensive battery of experiments starting from the real-world traffic flows recorded over the 8 crosses in the SUMMER-MUSTARD dataset. In order to evaluate the adaptation capabilities, we systematically introduced progressive magnitude disruptions over the initial 59 days of traffic flow data. Disruptions, such as accidents and adverse weather determine a decrease in the velocity which might create jams. Additionally, special activities such as sport events or beginning/end of holidays increase the flow magnitude. Such degenerated traffic conditions might happen due to non-recurrent events such as accidents, adverse weather or special events, such as football matches. Using the real-world flow and SUMO, we reproduce the traffic flow behavior when disruption occurs starting from normal traffic flow data by reflecting the disruption effect on vehicles speed and/or network capacity and demand. We sweep the disruption magnitude from normal traffic up to 5 levels of disruption reflected over all the 8 crosses over the entire day.

The evaluated systems are the following:

- BASELINE - is a optimized static traffic planning that uses pre-stored timing plans computed offline using historic data in the real-world.
- MILP - a Mixed-Integer Linear Programming phase plan optimization implementation inspired from [14].
- OSCILLATOR - A basic implementation of a network of Kuramoto oscillators ([15]) for each direction in the road network cross.
- OBELISC - uses the core Kuramoto oscillator model from [15] and considers an external reference for cycle time F, flow modulation k, and a spatial topology weight A. We used two implementations, one using an underlying ODE solver (OBELISC ODE) and the second one using NEF spiking neural networks (OBELISC NEF).
- Robust OBELISC - extends the basic OBELISC with the regularizing sliding mode control law u. The Robust OBELISC, similar to OBELISC, has two versions, Robust OBELISC ODE and Robust OBELISC NEF, respectively.

Evaluation of the phase calculation accuracy For the evaluation of the different approaches for phase duration computation (i.e. BASELINE, MILP, OSCILLATOR, OBELISC, and Robust OBELISC), we followed the next procedure:

- Read relevant data from simulation experiment (without disruptions and with 5 levels of progressive disruptions) for each of the systems.
- Compute relevant traffic aggregation metrics (i.e. average time loss, average speed, and average waiting time).
- Rank experiments depending on performance.
- Perform statistical tests (i.e. a combination of omnibus ANOVA and posthoc pairwise T-test with a significance $p = 0.05$) and adjust ranking depending on significance.
- Evaluate best algorithms depending on ranking for subsets of relevant metrics (i.e. the metrics with significant difference).

Our evaluation results are given in Table 1 where each of the approaches is ranked across the disruption magnitude scale (no disruption to maximum disruption) over the specific metrics (i.e. average time loss, and average speed, and waiting time, respectively). For flow magnitude disruptions, the level of disruption (i.e. 1.1 ... 1.5) is a factor used to adjust the number of vehicles or the speed of vehicles (i.e. for adverse weather) during the disruption. The evaluation was performed on the entire dataset containing recorded traffic flows over 59 days from 8 crosses. The exhaustive experiments and evaluation in Table 1 demonstrate where our

System/ Disruption level	normal flow	1.1	1.2	1.3	1.4	1.5
Average time loss(s)						
BASELINE	102.535	114.600	136.229	241.383	197.399	202.113
MILP	151.281	153.781	203.301	309.671	223.017	257.464
OSCILLATOR	131.468	161.871	203.301	309.671	199.797	497.124
OBELISC (ODE)	131.825	270.167	131.077	151.281	309.671	134.257
OBELISC (NEF)	135.355	155.782	153.524	200.265	199.357	216.919
Robust OBELISC (ODE)	133.524	143.904	147.524	153.524	200.265	220.008
Robust OBELISC (NEF)	85.726	88.326	89.726	84.165	89.889	84.291
*Average speed**						
BASELINE	5.81	5.67	5.46	5.02	4.94	4.75
MILP	5.97	5.87	5.46	5.03	4.92	4.61
OSCILLATOR	5.94	5.81	5.46	5.02	5.29	4.54
OBELISC (ODE)	5.97	4.75	5.14	5.22	5.04	5.11
OBELISC (NEF)	5.89	5.91	5.90	5.29	5.31	5.10
Robust OBELISC (ODE)	5.98	5.91	5.80	5.97	5.30	5.04
Robust OBELISC (NEF)	5.98	5.97	5.94	5.18	5.07	5.15
Waiting time(s)						
BASELINE	164.5	185.3	222.8	294.5	325.9	351.3
MILP	148.7	148.7	212.8	234.5	293.2	372.9
OSCILLATOR	115.7	142.2	215.8	286.5	208.5	418.3
OBELISC (ODE)	160.3	351.3	158.7	148.7	294.5	161.2
OBELISC (NEF)	137.1	137.6	139.4	216.0	204.2	236.3
Robust OBELISC (ODE)	139.4	141.4	149.4	169.8	216.8	252.5
Robust OBELISC (NEF)	128.7	145.7	148.8	159.2	162.4	158.7

$*$ Average speed calculated as the ratio between distance traveled and time of travel.

Table 1. Performance evaluation for the different phase duration calculation methods.

approach excels and where it fails to provide the best phase duration calculation. The chosen evaluation metrics reflect the overall performance (i.e. over multiple days) with respect to the most significant traffic metrics given the phase duration value computed by each of the systems.

The previous analysis is supported by the normalized ranking over the entire SUMMER-MUSTARD dataset in Figure 6, where we provide a condensed visual representation of each system's performance. Here, we can see that if we consider the average time loss the BASELINE performs worst due to its pre-defined timings and inability to adapt to unexpected disruptions during the daily traffic profile. At the other end of the ranking, both implementations of Robust OBELISC provide minimal waiting time capturing the fast and steep changes in the disrupted flows. Due to their similar core modelling and dynamics, OBELISC systems and OSCILLATOR tend to provide similar performance, with a relative improvement on the OBELISC side in terms of duration, waiting time, and speed metrics. This is due to its spatio-temporal extension beyond the basic oscillator model that can capture also the spatial contributions of adjacent flows beyond their temporal regularities when computing the phase duration. Looking at the various implementations of OBELISC, from the computational point of view, the NEF spiking neural networks excel in performance over the ODE versions due to their inherent learning and adaptation capabilities coupled with the distributed representations when solving the dynamics. Finally, the Robust OBELISC system provides overall superior performance through its discontinuous sliding mode control law that captures the deviation of the dynamics in the presence of disruptions and compensates robustly for their impact on the oscillator convergence (see Figure 5).

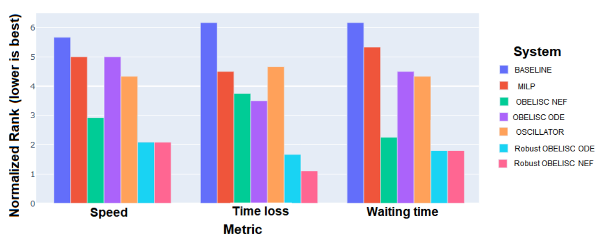

Fig. 6. Phase Duration Calculation System Ranking on All Metrics and Entire Dataset (8 crosses over 59 days).

Evaluation of the run-time In terms of run-time, the adaptive methods provide different levels of performance, mainly due to the modelling and optimization types they use. The BASELINE is excluded as it is just the static optimized plans allocation for the real traffic setting in SUMMER-MUSTARD, basically, a simple value recall from a look-up-table. We measured the time needed by each of the evaluated adaptive systems to provide a phase duration estimate after a sensory sample (i.e. one sensory reading of traffic flow data). As mentioned, each system uses a different computational approach: the MILP uses a solver that implements an LP-based branch-and-bound algorithm, the OSCILLATOR uses a Runge-Kutta 45 ODE solver, whereas OBELISC can be implemented using Runge-Kutta 45 ODE solver or connected populations of NEF spiking neural networks. The evaluation is given in Table 2 where the average value over the entire range of traffic conditions (normal and disruptions) is considered. The experimental setup for our experiments used 3 machines, each with 24 CPU cores and 132 GB RAM, and Apache Flink for stream processing and cluster management. As expected, at the level of a single intersection optimization, ODE solver

Model	Single cross	Region (8 crosses)
MILP	0.0510	0.3930
OSCILLATOR	0.0568	0.4544
Robust OBELISC ODE	0.0489	0.4534
Robust OBELISC NEF	**0.0071**	**0.0426**

Table 2. Adaptive phase duration calculation run-time evaluation.

approaches (i.e. MILP, OSCILLATOR, and OBELISC (ODE)) lie in the same range, providing a new phase duration value after 50 ms. At the region level, considering all 8 crosses, the run-time increases with an order of magnitude, with MILP overtaking the OSCILLATOR and OBELISC (ODE) due to MILP's constraint optimization efficiency at scale and the similar computations of OSCILLATOR and OBELISC (ODE). The fastest approach, both as single cross and regions level, is the NEF neural implementation of OBELISC. With more than 80% run-time improvement both at single cross-level and regional-level, OBELISC (NEF) excels due to its efficient computation and learning substrate.

4 Discussion

Traditionally, phase duration optimization for coordinated traffic signals is based on average travel times between intersections and average traffic volumes at each intersection.

Modelling Our study introduces an end-to-end modelling, control, and learning system for road traffic phase duration optimization applicable to any road traffic layout, scale, and architecture (i.e. number of lanes per direction etc.). More precisely, using an oscillator-based model [15] of the traffic flow dynamics in large signalized road networks, the system exploits the periodic nature of the traffic signal circular phasing similar to [1,5] - termed OSCILLATOR in our experiments. OBELISC goes beyond OSCILLATOR by considering a weighted external perturbation (e.g. cycle time reference weight F), flow modulation k, and a spatial topology weight A. Such a modelling approach adapts to unpredictable disruptions in traffic flows (e.g. accidents, re-routing, adverse weather conditions) up to a certain extent, where the dynamics of the disruption doesn't perturb the self-organization of the coupled oscillators.

Robust control In reality, the "steep derivatives" of traffic flows do not allow OBELISC and OSCILLATOR to converge to the best phase duration value. In order to achieve high performance (i.e. minimizing metrics such as time loss or maximizing average speed), we complemented OBELISC with a sliding mode controller. Such a robust controller "pushes" the perturbed dynamics under disruptions towards a dynamics that "drive" the coupled oscillators network towards the optimal phase. This way, the Robust OBELISC system is capable to solve local and global traffic dynamics by exploiting the coupling among different oscillators describing traffic periodicity under disruptions. The proposed system in [14] - termed MILP in our experiments - used exact mathematical programming techniques (i.e. mixed-integer linear programming) for optimizing the control of traffic signals and has shown only limited adaptation capabilities.

Computation and Learning Under real-world constraints of traffic control, OBELISC and Robust OBELISC cannot be implemented by simply integrating ODEs. To alleviate the typical convergence, stability, and robustness problems of ODE integration, we implemented OBELISC in efficient spiking neural networks using NEF. Basically, using distributed representations of traffic flow data, learning arbitrary functions from the data, and "compiling" the ODEs in neural populations, we gained efficient and flexible implementations of OBELISC (i.e. OBELISC NEF and Robust OBELISC NEF). Such a choice provided a clear advantage over the MILP implementation of [14] which formulated phase optimization into a continuous optimization problem without integer variables by modeling traffic flow as sinusoidal. The system solved a convex relaxation of the non-convex problem using a tree decomposition reduction and randomized rounding to recover a near-global solution. Given the complexity in expressing the system dynamics MILP performed well in simulations, yet the capability to adapt to sudden changes in the traffic situation are lacking (see the ranking in time loss, speed and waiting time in Figure 6).

5 Conclusions

Traffic control is a multi-dimensional problem to be optimized under deep uncertainty. Modelling traffic dynamics is fundamental for traffic control. Aiming at capturing the periodic nature of traffic, we propose OBELISC, a system using a network of oscillators capturing the spatial and temporal interactions among different crosses in a traffic network. In order to adaptively cope with unexpected traffic flow disruptions OBELISC is extended with a sliding mode controller that strengthens its adaptation capabilities towards global consensus under high-magnitude disruptions. The system is implemented as a lightweight learning system that exploits the coupling interactions among different controlled oscillators. Our extensive evaluation of the system on real-world data and against state-of-the-art methods, demonstrates the advantages OBELISC brings. From capturing the periodic dynamics of traffic phasing, to embedding the spatial correlation among traffic flow along its temporal dimensions, and up to robustly adapting to unexpected traffic disruptions, OBELISC stands out as a flexible solution for phase duration calculation. Finally, benefiting from efficient learning and computation in spiking neural networks, OBELISC is a strong candidate for actual real-world deployment.

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